The Dipole Moment

Note that the dipole solutions:

$$V(\bar{\mathrm{r}}) = \frac{Qd}{4\pi\varepsilon_0}\frac{\cos\theta}{r^2}$$

and

$$\mathbf{E}(\bar{\mathrm{r}}) = \frac{Qd}{4\pi\varepsilon_0}\frac{1}{r^3}\left[\ 2\cos\theta\ \hat{a}_r + \sin\theta\ \hat{a}_\theta\ \right]$$

provide the fields produced by an electric dipole that is:

1. **Centered** at the origin.

2. **Aligned** with the *z*-axis.

Q: *Well isn't that just grand. I suppose these equations are thus **completely useless** if the dipole is **not** centered at the origin and/or is **not** aligned with the z-axis !*!@!*

A: That is indeed **correct**! The expressions above are **only** valid for a dipole centered at the origin and aligned with the *z*-axis.

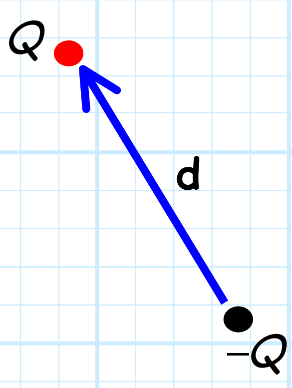
To determine the fields produced by a more **general case** (i.e., arbitrary location and alignment), we first need to **define** a new quantity **p**, called the **dipole moment**:

$$\mathbf{p} = Q\,\mathbf{d}$$

Note the dipole moment is a **vector** quantity, as the **d** is a vector quantity.

Q: *But what the heck is vector **d** ??*

A: Vector **d** is a **directed distance** that extends **from** the location of the **negative** charge, **to** the location of the **positive** charge. This directed distance vector **d** thus describes the **distance** between the dipole charges (vector magnitude), as well as the **orientation** of the charges (vector direction).



Therefore $\mathbf{d} = |\mathbf{d}|\,\hat{a}_d$, where:

$|\mathbf{d}|$ = **distance** d between charges

and

$\hat{a}_d$ = the **orientation** of the dipole

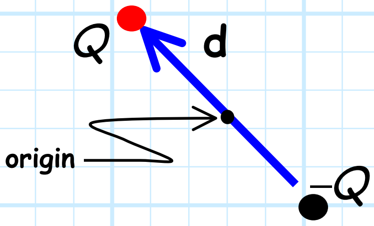
Note **if** the dipole is aligned with the z-axis, we find that $\mathbf{d} = d\ \hat{a}_z$. Thus, since $\hat{a}_z \cdot \hat{a}_r = \cos\theta$, we can write the expression:

$$\begin{aligned} Qd\ \cos\theta &= Q\ d\ \hat{a}_z \cdot \hat{a}_r \\ &= Q\ \mathbf{d} \cdot \hat{a}_r \\ &= \mathbf{p} \cdot \hat{a}_r \end{aligned}$$

Therefore, the electric potential field created by a dipole centered at the origin and aligned with the z-axis can be rewritten in terms of its dipole moment **p**:

$$\begin{aligned} V(\bar{r}) &= \frac{Qd}{4\pi\varepsilon_0} \frac{\cos\theta}{r^2} \\ &= \frac{1}{4\pi\varepsilon_0} \frac{\mathbf{p} \cdot \hat{a}_r}{r^2} \end{aligned}$$

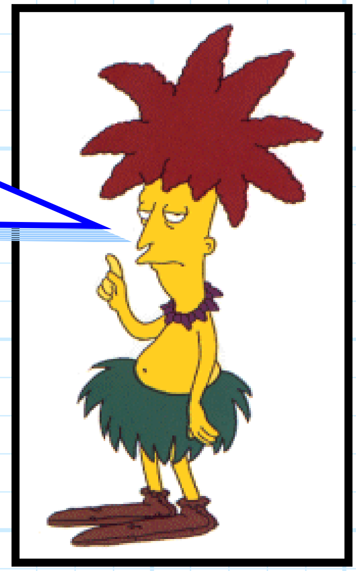
It turns out that, not **only** is this representation valid for a dipole aligned with the z-axis (e.g., $\mathbf{d} = d\,\hat{a}_z$), it is valid for electric dipoles located at the origin, and oriented in **any** direction!



$$V(\bar{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\mathbf{p} \cdot \hat{a}_r}{r^2}$$

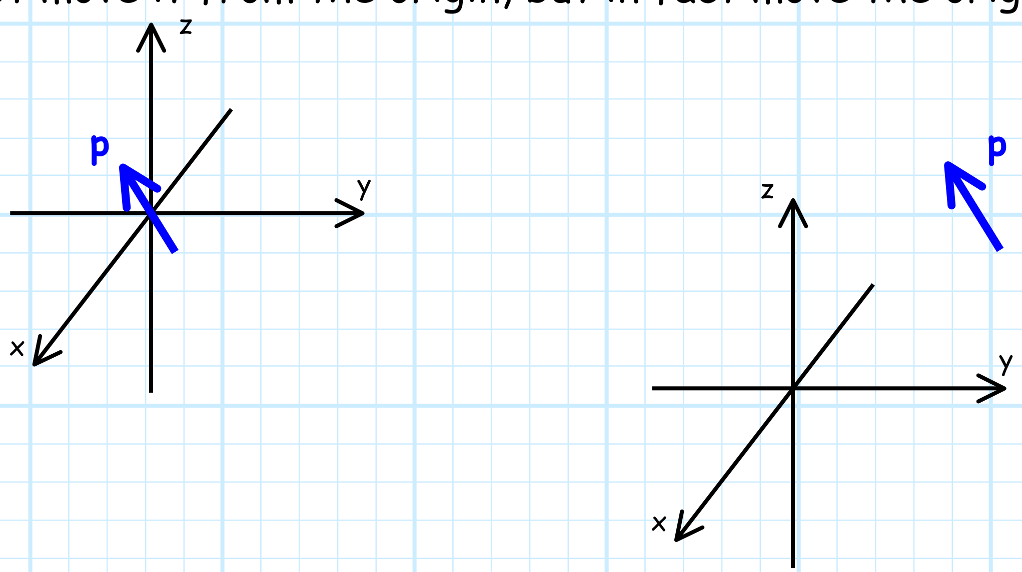
Although the expression above is valid for **any** and **all** dipole moments **p**, it is valid **only** for dipoles located at the origin (i.e., $\bar{r}' = 0$).

Q: ***Swell.*** *But you have* ***neglected*** *one significant detail—what are the fields produced by a dipole when it is* ***NOT*** *located at the origin?*

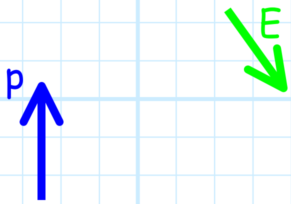


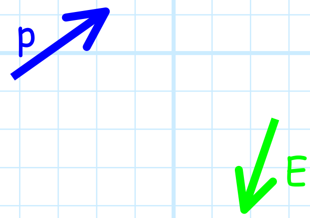
A: Finding the solution for **this** problem is our next task!

Note the electric dipole does **not** "know" where the origin is, or if it is located there. As far as the **dipole** is concerned, we do not move it from the origin, but in fact move the origin from **it**!

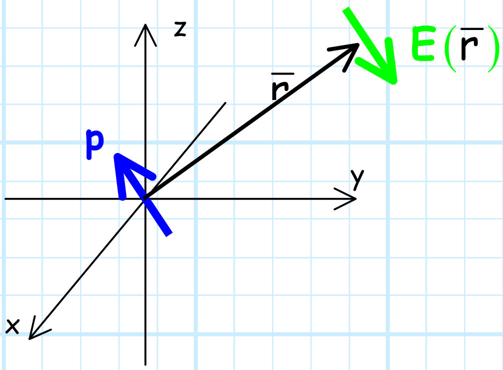


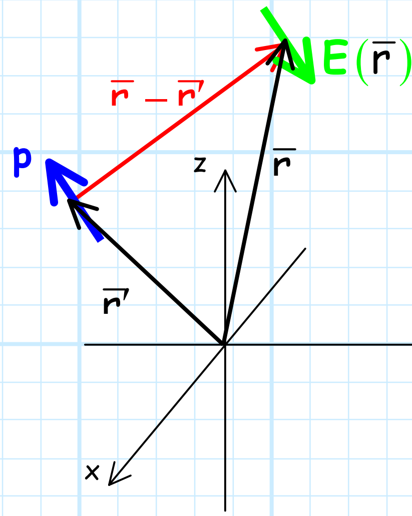
In other words, the fields produced by an electric dipole are **independent** of its location or orientation—it is the mathematics expressing these fields that get modified when we change our origin and coordinate system!

If: 

Then: 

Thus, we simply need to **translate** the previous field (dipole at the origin) solution by the **same** distance and direction that we move the dipole from the origin.





Just as with charge, the **location** of the dipole (center) is denoted by position vector $\bar{r}'$.

Note if the dipole is located at the origin, the position vector $\bar{r}$ extends from the dipole the location where we evaluate the electric field.

However, if the dipole is **not** located at the origin, this vector extending from the dipole to the electric field is **instead** $\bar{r} - \bar{r}'$. Thus, to translate the solution of the dipole at the origin to a new location, we replace vector $\bar{r}$ with vector $\bar{r} - \bar{r}'$, i.e.:

$$r = |\bar{r}| \qquad \text{becomes} \qquad |\bar{r} - \bar{r}'|$$

$$\hat{a}_r = \frac{\bar{r}}{|\bar{r}|} \qquad \text{becomes} \qquad \hat{a}_R = \frac{\bar{r} - \bar{r}'}{|\bar{r} - \bar{r}'|}$$

$$V(\bar{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\hat{a}_r}{r^2} \qquad \text{becomes} \qquad V(\bar{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}$$

Thus, a dipole of **any** arbitrary orientation and location produces the electric potential field:

$$\begin{aligned} V(\bar{r}) &= \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\hat{a}_R}{|\bar{r}-\bar{r}'|^2} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3} \end{aligned}$$